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A Mathematical Study on Non-Linear MHD Boundary Layer Past a Porous Shrinking Sheet with Suction

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Abstract

In this paper we discuss with magneto hydrodynamic viscous flow due to a shrinking sheet in the presence of suction. We also discuss two dimensional and axisymmetric shrinking for various cases. Using similarity transformation the governing boundary layer equations are converted into its dimensionless form. The transformed simultaneous ordinary differential equations are solved analytically by using Homotopy analysis method. The approximate analytical expression of the dimensionless velocity, dimensionless temperature and dimensionless concentration are derived using the Homotopy analysis method through the guessing solutions. Our analytical results are compared with the previous work and a good agreement is observed.

Keywords: Chemical reaction; Suction at the surface; Porous shrinking sheet' Non-linear ordinary differential equations; Homotopy analysis method.

1. Introduction

The flow over a shrinking surface is an important problem in many engineering processes with applications in industries. In nature, the presence of pure air or water is impossible. Some foreign mass may be present either naturally or mixed with the air or water.

Apelbat [1] investigated the mass transfer with a chemical raction of the first order effects of axial diffusion. Bhattacharyya et al. [2] examined the stability of viscous flow over a stretching sheet. Brady et al. [3] explained the steady flow in a channel or tube with accelerating surface velocity and found the exact solution to the Navier-Stokes equations with reverse flow. Cheng et al. [5] investigated the non-similarity solution and correlation of transient heat transfer in laminar boundary layer flow over a wedge. Gill [6] examined a process for the step-by-step integration of differential equations in an automatic digital computing machine. Crane [7] investigated the flow past a stretching plate. Gupta [8] et al. examined the heat and mass transfer on a stretching sheet with suction and blowing. Hakiem et al. [9] explained the joule heating effects on MHD free convection flow of a micro polar fluid. Hayat et al. [10] found the analytic solution of magnetohydrodynamic flow of a second grade fluid over a shrinking sheet.

Magnetohydrodynamic (MHD) mixed convection heat transfer flow in porous and non-porous media is of considerable interest in the technical field due to its frequent occurrence in industrial technology and geothermal application. Jensen et al. [11] investigated the flow phenomena in chemical vapor deposition of thin films. Kuo Bor-

Lin [12] examined the heat transfer analysis for the Falkner-Skan wedge flow by the differential transformation method. Miklavcic et al. [13] explained the viscous flow due to a shrinking sheet. Muhaimin et al.[14] investigated the effects of heat and mass transfer on nonlinear MHD boundary layer flow over a shrinking sheet in the presence of suction. Sajid et al.[15] examined the MHD rotating flow of viscous fluid over a shrinking surface.

Sajid et al.[16] explained the application of Homotopy analysis method for MHD viscous flow due to a shrinking sheet. Troy [18] found uniqueness of flow of a second-order fluid past a stretching sheet. Usha et al.1[29] investigated the axisymmetric motion of a liquid film on an unsteady stretching surface. Wang et al.[30] examined the fluid flow due to a stretching cylinder.

The aim of this paper is to discuss the mathematical analysis of non-linear MHD boundary layer past a porous shrinking sheet with suction. The approximate analytical expressions of the dimensionless velocity profiles, dimensionless temperature profiles, and dimensionless concentration profiles are derived using the Homotopy analysis method and discussed by graphically.

2. Mathematical formulation of the problem

Let us consider the MHD flow of an incompressible viscous fluid over a shrinking sheet at $y = 0$. The x and y axes are taken along and perpendicular to the sheet respectively, as shown in Fig.1. The fluid is assumed to be Newtonian and electrically conducting and the flow is confined to $y > 0$. A constant magnetic field of strength B acts in the direction of y axis. The induced magnetic field is negligible, which is a valid assumption on a laboratory scale. The assumption is justified when the magnetic Reynolds number is small, Hayat et al. (2007). Since no electric field is applied and the effect of polarization of the ionized fluid is negligible, we can assume that the electric field $E = 0$.

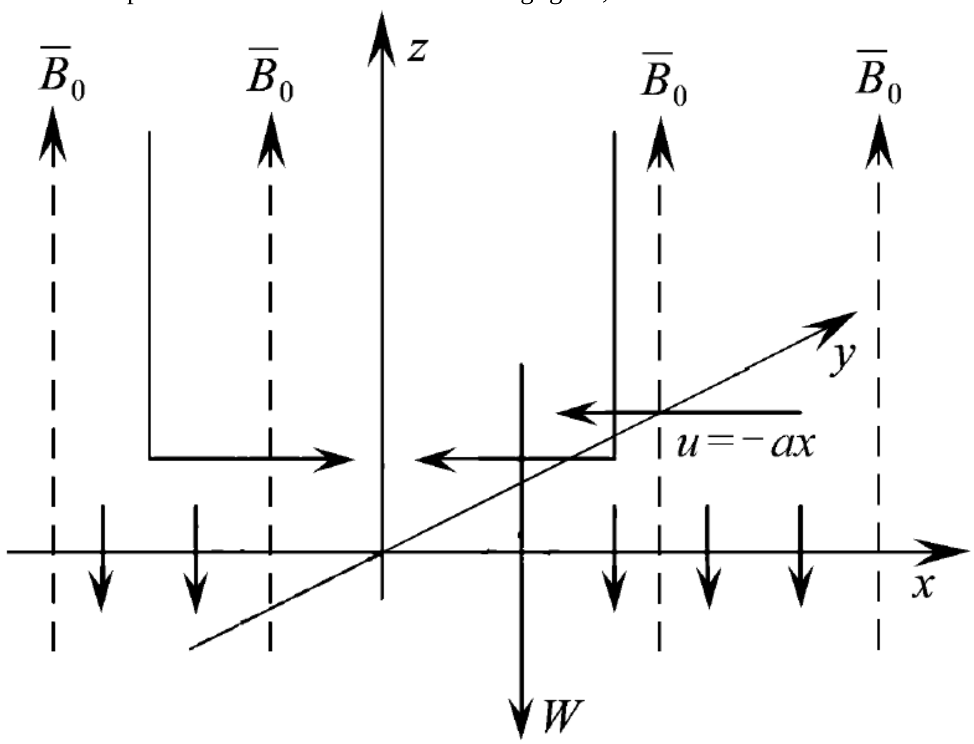

Fig.1 Schematic diagram of fluid flow problem

The chemical reactions are taking place in the flow and a constant suction is imposed at the horizontal surface, see Fig.1. The governing boundary layer equations of momentum, energy and diffusion for the MHD flow in terms of vector notation are defined as follows:
Continuity equation:

$$div\,\vec{V} = 0 \tag{1}$$

Momentum equation:

$$(\vec{V}.grad\ \vec{V}) = -\frac{1}{\rho} grad\ p + \nu\nabla^2\vec{V} + \frac{1}{\rho}\vec{j}\times\vec{B} \tag{2}$$

d Energy equation:

$$(\vec{V}.grad)T = \frac{k_e}{\rho c_p}\nabla^2 T \tag{3}$$

Species concentration equation:

$$(\vec{V}.grad)C = D\nabla^2 C \pm k_1 C \tag{4}$$

Where $\vec{j} = \sigma(\vec{E} + \vec{V}\times\vec{B} - \frac{1}{en_e} grad\ p_e\ div\,\vec{B} = 0, curl\ \vec{H} = 0\ and\ curl\,\vec{E} = 0$

Where $\vec{V}$ is the velocity vector, p is the pressure, ν is the kinematic coefficient of viscosity.
Continuity equation in terms of vector notation is

$$\frac{\partial\rho}{\partial t} + \nabla.(\rho\vec{V}) = 0.$$

For steady incompressible flow:

$\frac{\partial\rho}{\partial t} = 0$ and ρ is a constant.

Continuity equation becomes

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0.$$

Finally, the continuity equation is reduced to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

Under these conditions, the basic governing boundary layer equations of momentum, energy and diffusion for mixed convection flow neglecting Joule's viscous dissipation can be simplified to the following equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \text{ (continuity)} \tag{5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) - \frac{\sigma B_0{}^2}{\rho}u - \frac{\nu}{K}u \quad (x\text{-Momentum}) \tag{6}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) - \frac{\sigma B_0{}^2}{\rho}v - \frac{\nu}{K}v \quad (y\text{-Momentum}) \tag{7}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) \quad (z\text{-Momentum}) \tag{8}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}) \quad \text{(Energy)} \tag{9}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}) - k_1 C \quad \text{(Diffusion)} \tag{10}$$

Where *u, v, w* are the velocity components in the *x, y* and *z* directions respectively. ν is the kinematic viscosity, *p* is the pressure, σ is the electrical conductivity, ρ is the density of the fluid, B_0 is the magnetic induction, α is the thermal conductivity of the fluid, μ is the dynamic viscosity, *K* is the porous medium permeability, $c_p g$ is the specific heat at constant pressure and k_1 is the rate of chemical reaction.

The boundary conditions applicable to the present flow are

$$u = -U = -ax,\ v = -a\,(m-1)y,$$
$$w = -W,\ T = T_w,\ C = C_w\ \ at\ \ y = 0, \tag{11}$$
$$u \to 0,\ T \to T_\infty,\ C \to C_\infty\ \ as\ y \to \infty$$

in which $a > 0$ is the shrinking constant, $W > 0$ is the suction velocity, $m = 1$ when sheet shrinks in x-direction only and $m = 2$ when the sheet shrinks axsymmetrically.

Introducing the following similarity transformations

$$u = a\,x\,f^{'}(\eta),\ v = a(m-1)y\,f^{'}(\eta),\ w = -\sqrt{a\,\nu\,m}\,f(\eta),\ \eta = \sqrt{\frac{a}{\nu}}z,$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \quad and \quad \phi = \frac{C - C_\infty}{C_w - C_\infty} \tag{12}$$

The eqn. (1) is identically satisfied and the eqn. (8) can be integrated to give

$$\frac{p}{\rho} - \nu\frac{\partial w}{\partial z} - \frac{w^2}{2} + \text{constant} \tag{13}$$

The eqns. (6)-(11) reduces to the following boundary value problem

$$f^{'''} - (M^2 + \Pr\lambda)f^{'} - f^{'2} + m\,f\,f^{''} = 0 \tag{14}$$

$$\theta^{''} + m\Pr f\,\theta^{'} - \Pr\theta\,f^{'} = 0 \tag{15}$$

$$\phi^{''} - Sc\,f^{'}\phi + m\,sc\,f\,\phi^{'} - Sc\gamma\,\phi = 0 \tag{16}$$

The boundary conditions can be written as

$$f(0) = S,\ f^{'}(0) = -1,\ \theta(0) = 1,\ \phi(0) = 1 \quad at\,\eta = 0$$
$$f^{'}(\infty) = 0,\ \theta(\infty) = 0,\ \phi(\infty) = 0\ \ at\ \eta \to \infty \tag{17}$$

Where *Pr* is the Prandtl number, *Sc* is the Schmidt number, M^2 is the Magnetic parameter, γ is the Chemical reaction parameter, λ porosity parameter and S is the suction parameter can be defined as follows:

$$\Pr = \frac{\nu}{\alpha},\ Sc = \frac{\nu}{D},\ M^2 = \frac{\sigma B_0^{\,2}}{\rho a},\ \gamma = \frac{k_1}{a},\ \lambda = \frac{\alpha}{a\,K}\ and\ \ S = \frac{W}{m\sqrt{a\,\nu}} \tag{18}$$

when $m = 1$ (sheet shrinks in x-direction) and $m = 2$ (sheet shrinks in axisymmetrically). The mass diffusion equation (16) can be adjusted to meet these circumstances if one takes $\gamma > 0$ for destructive reaction, $\gamma = 0$ for no reaction and $\gamma < 0$ for generative reaction.

3. Solution of the problem using the Homotopy analysis method

This section deals with a basic strong analytic tool for non-linear problems, namely the Homotopy analysis method (HAM) which was generated by Liao [19], is employed to solve the nonlinear differential eqns. (14) – (16). The Homotopy analysis method is based on a basic concept in topology. Unlike perturbation techniques like [20], the Homotopy analysis method is independent of the small/large parameters. Unlike all other reported perturbation and non-perturbation techniques such as the artificial small parameter method [21], the δ -expansion method [22] and Adomian's decomposition method [23], the Homotopy analysis method provides us a simple way to adjust and control the convergence region and rate of approximation series. The Homotopy analysis method has been successfully applied to many nonlinear problems such as heat transfer [24], viscous flows [25], nonlinear oscillations [26], Thomas-Fermi's atom model [27], nonlinear water waves [28], etc. Such varied successful applications of the Homotopy analysis method confirm its validity for nonlinear problems in science and engineering. The Homotopy analysis method is a good technique when compared to other perturbation methods. The existence of the auxiliary parameter h in the Homotopy analysis method provides us with a simple way to adjust and control the convergence region of the solution series.

3.1 Basic concepts of the Homotopy analysis method

Consider the following differential equation:

$$N[u(t)] = 0 \tag{19}$$

Where N is a nonlinear operater, t denotes an independent variable, $u(t)$ is an unknown function. For simplicity, weignore all boundary or initial conditions, which can be treated in the similar way. By means of generalizing the conventional Homotopy method, Liao constructed the so-called zero-order deformation equation as:

$$(1-p)L[\varphi(t;p)-u_0(t)] = p\,h\,H(t)N[\varphi(t;p)] \tag{20}$$

Where $p \in [0,1]$ is the embedding parameter, $h \neq 0$ is a nonzero auxiliary parameter, $H(t) \neq 0$ is an auxiliary function, L an auxiliary linear operater, $u_0(t)$ is an initial guess of $u(t), \varphi(t;p)$ is an unknown function. It is important to note that one has great freedom to choose auxiliary unknowns in HAM. Obviously, when p = 0 and p =1, it holds:

$$\varphi(t;0) = u_0(t) \; and \; \varphi(t;1) = u(t) \tag{21}$$

Respectively. Thus, as p increases from 0 to 1, the solution $\varphi(t;p)$ varies from the initial guess $u_0(t)$ to the solution $u(t)$.

Expanding $\varphi(t;p)$ in Taylor series with respect to p , we have:

$$\varphi(t;p) = u_0(t) + \sum_{m=1}^{+\infty} u_m(t)p^m \tag{22}$$

$$u_m(t) = \frac{1}{m!}\frac{\partial^m \varphi(t;p)}{\partial p^m} \tag{23}$$

If the auxiliary linear operator, the initial guess, the auxiliary parameter h, and the auxiliary function are so properly chosen, the series eqn.(22) converges at $p = 1$ then we have:

$$u(t) = u_0(t) + \sum_{m=1}^{+\infty} u_m(t) \tag{24}$$

Differentiating the eqn.(20) for m times with respect to the embedding parameter p , and then setting $p = 0$ and finally dividing them by m!, we will have so-called mth order deformation equation as:

$$L[u_m - \chi_m u_{m-1}] = h\,H(t)\Re_m\left(\underset{m-1}{\vec{u}}\right) \tag{25}$$

Where

$$\Re_m\left(\underset{m-1}{\vec{u}}\right) = \frac{1}{(m-1)!}\frac{\partial^{m-1} N[\varphi(t:p)]}{\partial p^{m-1}} \tag{26}$$

And

$$\chi_m = \begin{cases} 0, \; m \leq 1, \\ 1, \; m > 1. \end{cases} \tag{27}$$

Applying L^{-1} on both side of equation.(25), we get

$$u_m(t) = \chi_m u_{m-1}(t) + h\, L^{-1}\left[H(t)\Re_m\left(\vec{u}_{m-1}\right)\right] \tag{28}$$

In this way, it is easily to obtain u_m for $m \geq 1$, at M^{th} order, we have

$$u(t) = \sum_{m=0}^{M} u_m(t) \tag{29}$$

When $M \to +\infty$, we get an accurate approximation of the original eqn.(19). For the convergence of the above method we refer the reader to Liao [19]. If equation.(19) admits unique solution, then this method will produce the unique solution.

4. Approximate analytical expressions of the non-linear differential eqns.(14) - (17) using Homotopy analysis method

$$f^{'''} - (M^2 + \Pr\lambda) f^{'} - f^{'2} + m\, f\, f^{''} = 0 \tag{30}$$

$$\theta^{''} + m \Pr f\, \theta^{'} - \Pr \theta\, f^{'} = 0 \tag{31}$$

$$\phi^{''} - Sc\, f^{'}\phi + m\, sc\, f\, \phi^{'} - Sc\gamma\, \phi = 0 \tag{32}$$

We construct the Homotopy for the eqns.(30),(31) and (32) are as follows:

$$(1-p)\left(\frac{d^3 f}{d\eta^3} - \left(M^2 + \Pr\lambda\right)\frac{df}{d\eta}\right) + h\, p\left(\frac{d^3 f}{d\eta^3} - \left(M^2 + \Pr\lambda\right)\frac{df}{d\eta} - \left(\frac{df}{d\eta}\right)^2 + mf\,\frac{d^2 f}{d\eta^2}\right) = 0 \tag{33}$$

$$(1-p)\frac{d^2\theta}{d\eta^2} + h\, p\left(\frac{d^2\theta}{d\eta^2} + m \Pr f\,\frac{d\theta}{d\eta} - \Pr\theta\,\frac{df}{d\eta}\right) = 0 \tag{34}$$

$$(1-p)\left(\frac{d^2\phi}{d\eta^2} - Sc\,\gamma\,\phi\right) + h\, p\left(\frac{d^2\phi}{d\eta^2} + m Sc\, f\,\frac{d\phi}{d\eta} - Sc\,\gamma\,\phi\right) = 0 \tag{35}$$

The approximate solution of the eqns.(33),(34) and (35) are as follows:

$$f = f_0 + pf_1 + p^2 f_2 + p^3 f_3 + \dots\dots\dots \tag{36}$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + p^3\theta_3 + \dots\dots \tag{37}$$

$$\phi = \phi_0 + p\phi_1 + p^2\phi_2 + p^3\phi_3 + \dots\dots \tag{38}$$

The initial approximations are as follows:

$$f_0(0) = S\ ,\ f_0^{'}(0) = -1\ ,\ \theta_0(0) = 1\ ,\ \phi_0(0) = 1 \tag{39}$$

$$f_i(0) = 0\ ,\ f_i^{'}(0) = 0\ ,\ \theta_i(0) = 0\ ,\ \phi_i(0) = 0 \quad for\, i = 1,2,3.\dots \tag{40}$$

$$f_0^{'}(\infty) = 0\ ,\ \theta_0(\infty) = 0\ ,\ \phi_0(\infty) = 0 \tag{41}$$

$$f_i^{'}(\infty) = 0\ ,\ \theta_i(\infty) = 0\ ,\ \phi_i(\infty) = 0 \quad for\, i = 1,2,3.\dots\dots \tag{42}$$

Substituting the eqns.(36) ,(37) and (38) into the eqns.(33),(34) and (35) respectively we get

$$(1-p)\left(\frac{d^3}{d\eta^3}(f_0+pf_1+\ldots)-\left(M^2+\Pr\lambda\right)\frac{d}{d\eta}(f_0+pf_1+\ldots.)\right)$$
$$+h\,p\left(\begin{array}{l}\frac{d^3}{d\eta^3}(f_0+pf_1+\ldots..)-\left(M^2+\Pr\lambda\right)\frac{d}{d\eta}(f_0+pf_1+\ldots..)\\ -\left(\frac{d}{d\eta}(f_0+pf_1+\ldots..)\right)^2+mf\,\frac{d^2}{d\eta^2}(f_0+pf_1+\ldots.)\end{array}\right)=0 \tag{43}$$

$$(1-p)\frac{d^2}{d\eta^2}(\theta_0+p\theta_1+\ldots.)$$
$$+h\,p\left(\begin{array}{l}\frac{d^2}{d\eta^2}(\theta_0+p\theta_1+\ldots..)+m\Pr f\,\frac{d}{d\eta}(\theta_0+p\theta_1+\ldots.)\\ -\Pr\theta\frac{d}{d\eta}(\theta_0+p\theta_1+\ldots...)\end{array}\right)=0 \tag{44}$$

$$(1-p)\left(\frac{d^2}{d\eta^2}(\phi_0+p\phi_1+\ldots.)-Sc\,\gamma(\phi_0+p\phi_1+\ldots.)\right)$$
$$+h\,p\left(\begin{array}{l}\frac{d^2}{d\eta^2}(\phi_0+p\phi_1+\ldots..)+m\,Sc\,f\,\frac{d}{d\eta}(\phi_0+p\phi_1+\ldots...)\\ -Sc\,\gamma(\phi_0+p\phi_1+\ldots..)\end{array}\right)=0 \tag{45}$$

Comparing the coefficients of p^0, p^1 in the eqns.(43),(44) and (45), respectively we get

$$p^0\ :\ \frac{d^3 f_0}{d\eta^3}-\left(M^2+\Pr\lambda\right)\frac{df_0}{d\eta}=0 \tag{46}$$

$$p^1\ :\ \frac{d^3 f_1}{d\eta^3}-\frac{d^3 f_0}{d\eta^3}-\left(M^2+\Pr\lambda\right)\left(\frac{df_0}{d\eta}-\frac{df_1}{d\eta}\right)$$
$$+h\left(\frac{d^3 f_0}{d\eta^3}-\left(M^2+\Pr\lambda\right)\frac{df_0}{d\eta}-\left(\frac{df_0}{d\eta}\right)^2+mf_0\,\frac{d^2 f_0}{d\eta^2}\right)=0 \tag{47}$$

$$p^0\ :\frac{d^2\theta_0}{d\eta^2}=0 \tag{48}$$

$$p^1\ :\ \frac{d^2\theta_1}{d\eta^2}-\frac{d^2\theta_0}{d\eta^2}+h\left(\frac{d^2\theta_0}{d\eta^2}+m\Pr f_0\,\frac{d\theta_0}{d\eta}-\Pr\theta_0\,\frac{d\theta_0}{d\eta}\right)=0 \tag{49}$$

$$p^0\ :\frac{d^2\phi_0}{d\eta^2}-Sc\,\gamma\,\phi_0=0 \tag{50}$$

$$p^1\ :\ \frac{d^2\phi_1}{d\eta^2}-\frac{d^2\phi_0}{d\eta^2}+Sc\,\gamma\,\phi_0+h\left(\frac{d^2\phi_0}{d\eta^2}+m\,Sc\,f_0\,\frac{d\phi_0}{d\eta}-Sc\,\gamma\,\phi_0=0\right) \tag{51}$$

Let the initial solution of the eqns.(46), (48) and (50) using (39) and (41) are as follows:

$$f_0' = -e^{-\frac{a\eta}{S}} \tag{52}$$

$$\theta_0 = e^{-m\Pr\eta} \tag{53}$$

$$\phi_0 = e^{-m\,Sc\,\gamma\,\eta} \tag{54}$$

Solving the eqns.(47), (49) and (51) using (40) and (42) we get the following results

$$f_1' = \frac{\left(M^2+\Pr\lambda\right)S^2 e^{-\frac{a\eta}{S}}}{a^2} - \frac{S^2 e^{-\frac{2a\eta}{S}}}{a^2} + ma\left(\frac{S^2 e^{-\frac{a\eta}{S}}}{a^2} - \frac{S^2 e^{-\frac{a\eta}{S}}}{a^3} + \frac{S^2 e^{-\frac{2a\eta}{S}}}{4a^3}\right)$$
$$+1 - \frac{\left(M^2+\Pr\lambda\right)S^2}{a^2} + \frac{S^2}{4a^2} - ma\left(\frac{S^2}{a^2} - \frac{3S^2}{4a^3}\right) \tag{55}$$

$$\theta_1 = m^2\Pr^2\left(\frac{S e^{-m\Pr\eta}}{m^2\Pr^2} - \frac{S e^{-m\Pr\eta}}{a m^2 pr^2} + \frac{S e^{-\left(\frac{a}{S}+m\Pr\right)\eta}}{a\left(\frac{a}{S}+m\Pr\right)^2}\right) + \frac{\Pr e^{-\left(\frac{a}{S}+m\Pr\right)\eta}}{\left(\frac{a}{S}+m\Pr\right)^2}$$
$$+1 - m^2\Pr^2\left(\frac{S}{m^2\Pr^2} - \frac{S}{a m^2\Pr^2} + \frac{S}{a\left(\frac{a}{S}+m\Pr\right)^2}\right) - \frac{\Pr}{\left(\frac{a}{S}+m\Pr\right)^2} \tag{56}$$

$$\phi_1 = \frac{-Sc\, e^{-\left(mSc\gamma+\frac{a}{S}\right)\eta}}{\left(mSc\gamma+\frac{a}{S}\right)^2} + m^2Sc^2\gamma\left(\frac{S e^{-mSc\gamma\eta}}{m^2Sc^2\gamma^2} - \frac{S e^{-mSc\gamma\eta}}{a m^2Sc^2\gamma^2} + \frac{S e^{-\left(mSc\gamma+\frac{a}{S}\right)\eta}}{a\left(mSc\gamma+\frac{a}{S}\right)^2}\right) + \frac{e^{-mSc\gamma\eta}}{Sc\gamma m^2}$$
$$+1 + \frac{Sc}{\left(mSc\gamma+\frac{a}{S}\right)^2} - m^2Sc^2\gamma\left(\frac{S}{m^2Sc^2\gamma^2} - \frac{S}{a m^2Sc^2\gamma^2} + \frac{S}{a\left(mSc\gamma+\frac{a}{S}\right)^2}\right) - \frac{1}{Sc\gamma m^2} \tag{57}$$

Where $a = m\,Sc\,\gamma\,\lambda\Pr S$

According to the HAM, we can conclude that

$$f' = \lim_{p\to1} f(y) = f_0' + f_1' \tag{58}$$

$$\theta = \lim_{p\to1}\theta(y) = \theta_0 + \theta_1 \tag{59}$$

$$\phi = \lim_{p\to1}\phi(y) = \phi_0 + \phi_1 \tag{60}$$

Substituting the eqns.(52) and (55) in an eqn.(58) and (53) and (56) in an eqn.(59) and (54)and (57) in an eqn.(60) respectively we get the following results.

$$f' = -e^{-\frac{a\eta}{S}} + h\left(\begin{array}{l} \dfrac{(M^2+\Pr\lambda)S^2 e^{-\frac{a\eta}{S}}}{a^2} - \dfrac{S^2 e^{-\frac{2a\eta}{S}}}{a^2} \\ + m\,a\left(\dfrac{S^2 e^{-\frac{a\eta}{S}}}{a^2} - \dfrac{S^2 e^{-\frac{a\eta}{S}}}{a^3} + \dfrac{S^2 e^{-\frac{2a\eta}{S}}}{4a^3}\right) \\ +1 - \dfrac{(M^2+\Pr\lambda)S^2}{a^2} + \dfrac{S^2}{4a^2} - m\,a\left(\dfrac{S^2}{a^2} - \dfrac{3S^2}{4a^3}\right) \end{array}\right) \tag{61}$$

$$\theta = e^{-m\Pr\eta} + h\left(\begin{array}{l} m^2\Pr^2\left(\dfrac{S\,e^{-m\Pr\eta}}{m^2\Pr^2} - \dfrac{S\,e^{-m\Pr\eta}}{a\,m^2\,pr^2} + \dfrac{S\,e^{-\left(\frac{a}{S}+m\Pr\right)\eta}}{a\left(\frac{a}{S}+m\Pr\right)^2}\right) + \dfrac{\Pr e^{-\left(\frac{a}{S}+m\Pr\right)\eta}}{\left(\frac{a}{S}+m\Pr\right)^2} \\ +1 - m^2\Pr^2\left(\dfrac{S}{m^2\Pr^2} - \dfrac{S}{a\,m^2\Pr^2} + \dfrac{S}{a\left(\frac{a}{S}+m\Pr\right)^2}\right) - \dfrac{\Pr}{\left(\frac{a}{S}+m\Pr\right)^2} \end{array}\right) \tag{62}$$

$$\phi = e^{-m\,Sc\,\gamma\eta} + h\left(\begin{array}{l} \dfrac{-Sc\,e^{-\left(m\,Sc\,\gamma+\frac{a}{S}\right)\eta}}{\left(m\,Sc\,\gamma+\frac{a}{S}\right)^2} \\ + m^2Sc^2\gamma\left(\dfrac{S\,e^{-m\,Sc\gamma\,\eta}}{m^2Sc^2\gamma^2} - \dfrac{S\,e^{-m\,Sc\gamma\,\eta}}{a\,m^2Sc^2\gamma^2} + \dfrac{S\,e^{-\left(m\,Sc\,\gamma+\frac{a}{S}\right)\eta}}{a\left(m\,Sc\,\gamma+\frac{a}{S}\right)^2}\right) + \dfrac{e^{-m\,Sc\,\gamma\,\eta}}{Sc\gamma\,m^2} \\ +1 + \dfrac{Sc}{\left(m\,Sc\,\gamma+\frac{a}{S}\right)^2} - m^2Sc^2\gamma\left(\dfrac{S}{m^2Sc^2\gamma^2} - \dfrac{S}{a\,m^2Sc^2\gamma^2} + \dfrac{S}{a\left(m\,Sc\,\gamma+\frac{a}{S}\right)^2}\right) \\ - \dfrac{1}{Sc\gamma\,m^2} \end{array}\right) \tag{63}$$

5. Results and discussion

Figure 1 shows geometry of the problem. Figure 2-4 represents dimensionless temperature $\theta(\eta)$ versus dimensionless distance η. From Fig.2, it is noted that when the chemical reaction parameter γ increases, the dimensionless temperature profiles remains constant in some fixed values of the other dimensionless parameters .

From Fig.3, it is inferred that when the magnetic parameter M^2 increases the temperature profiles remains constant in some fixed values of the other dimensionless parameters. From Fig.4, it depicts that when the sheet shrinks parameter *m increases* the corresponding dimensionless temperature profiles decreases, in some fixed values of the other dimensionless parameters.

Figure 5-6 represents the dimensionless velocity $f^{'}(\eta)$ versus the dimensionless distance η. From Fig.5, it is noted that when the magnetic parameter M^2 increases, the corresponding dimensionless velocity also increases in some fixed values of the other dimensionless parameters. From Fig.6, it is inferred that when the sheet shrinks parameter m increases the corresponding dimensionless velocity profiles also increases in some fixed values of the other dimensionless parameters.

Figure 7-9 represents the dimensionless concentration $\phi(\eta)$ versus the dimensionless distance η. From Fig.7, it depicts that when the chemical reaction parameter γ increases the corresponding dimensionless concentration profiles decreases, in some fixed values of the other dimensionless parameters. From Fig.8, it is noted that when the magnetic parameter M^2 increases, the corresponding dimensionless concentration profiles decreases in some fixed values of the other dimensionless parameters. From Fig.9, it is inferred that when the Sheet shrinks parameter m increases the corresponding concentration profiles decreases in some fixed values of the other dimensionless parameters.

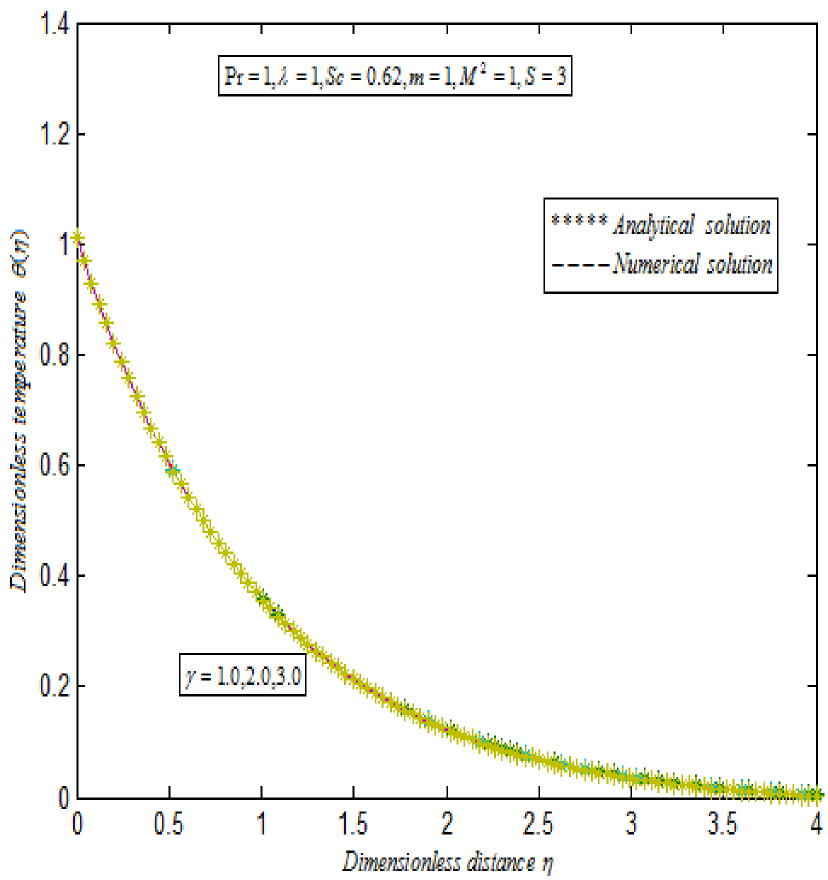

Fig.2: Dimensionless temperature $\theta(\eta)$ versus the dimensionless distance η . The curves are plotted using the eqn.(62) for various values of the chemical reaction parameter γ, and in some fixed values of the other dimensionless parameters $\Pr, \lambda, Sc, m, M^2, S.$

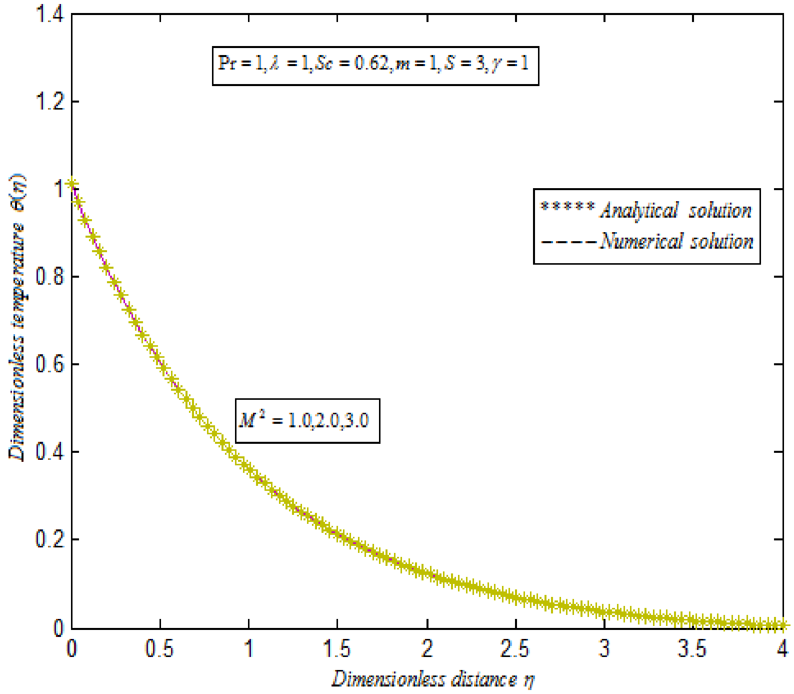

Fig.3: Dimensionless temperature $\theta(\eta)$ versus the dimensionless distance η . The curves are plotted using the eqn.(62) for various values of the magnetic parameter M^2, and in some fixed values of the other dimensionless parameters $\Pr, \lambda, Sc, m, \gamma, S$.

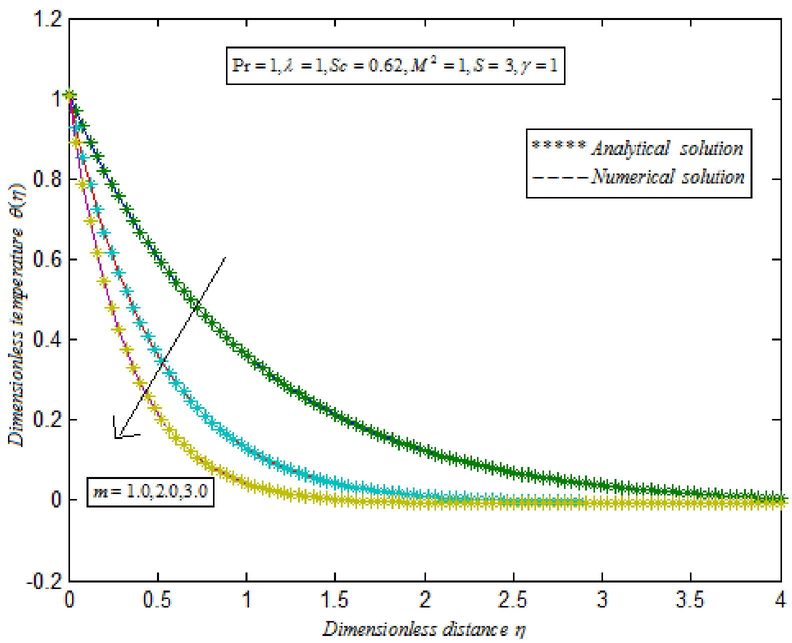

Fig.4: Dimensionless temperature $\theta(\eta)$ versus the dimensionless distance η . The curves are plotted using the eqn.(62) for various values of the Sheet shrinks parameter m, and in some fixed values of the other dimensionless parameters $\Pr, \lambda, Sc, M^2, \gamma, S$.

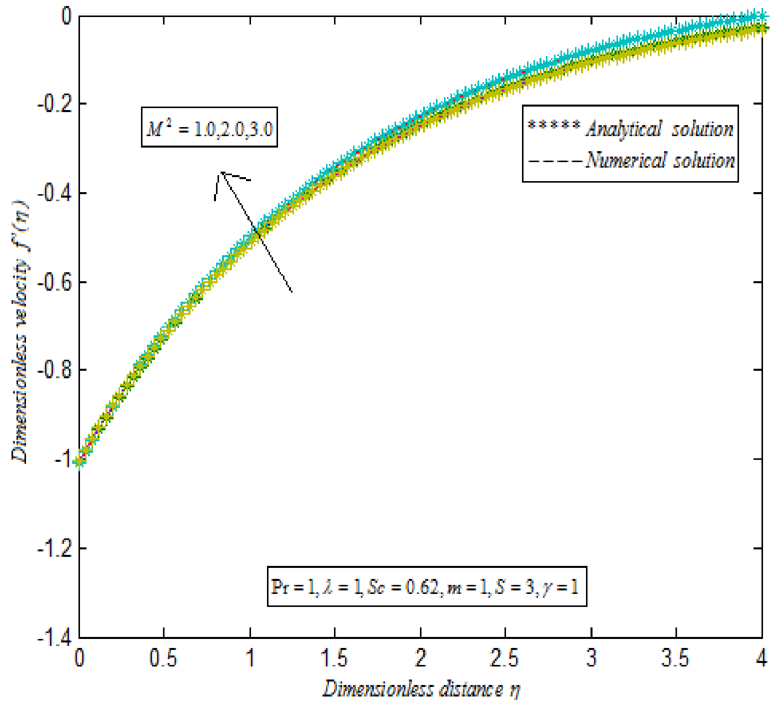

Fig.5: Dimensionless velocity $f'(\eta)$ versus the dimensionless distance η . The curves are plotted using the eqn.(61) for various values of the Magnetic parameter M^2, and in some fixed values of the other dimensionless parameters $\Pr, \lambda, Sc, m, \gamma, S.$

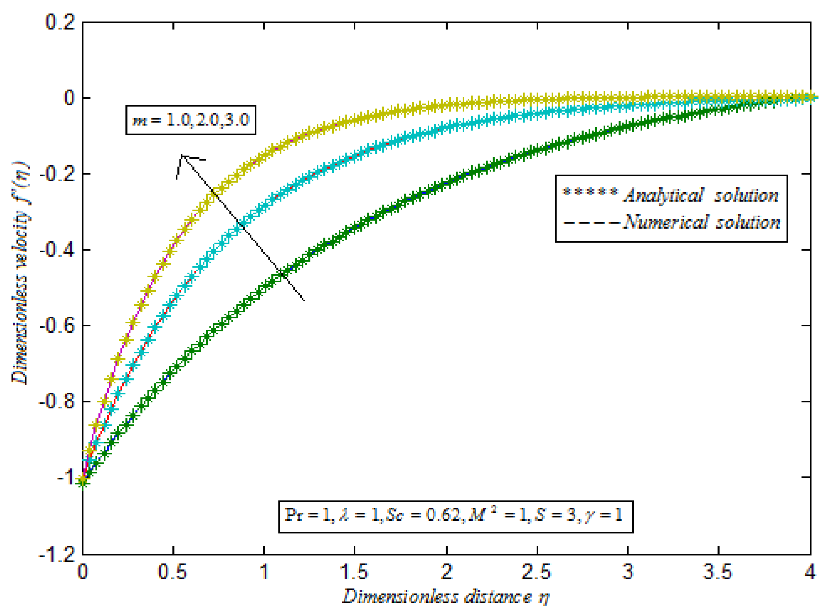

Fig.6: Dimensionless velocity $f'(\eta)$ versus the dimensionless distance η . The curves are plotted using the eqn.(61) for various values of the Sheet shrinks parameter m, and in some fixed values of the other dimensionless parameters $\Pr, \lambda, Sc, M^2, \gamma, S.$

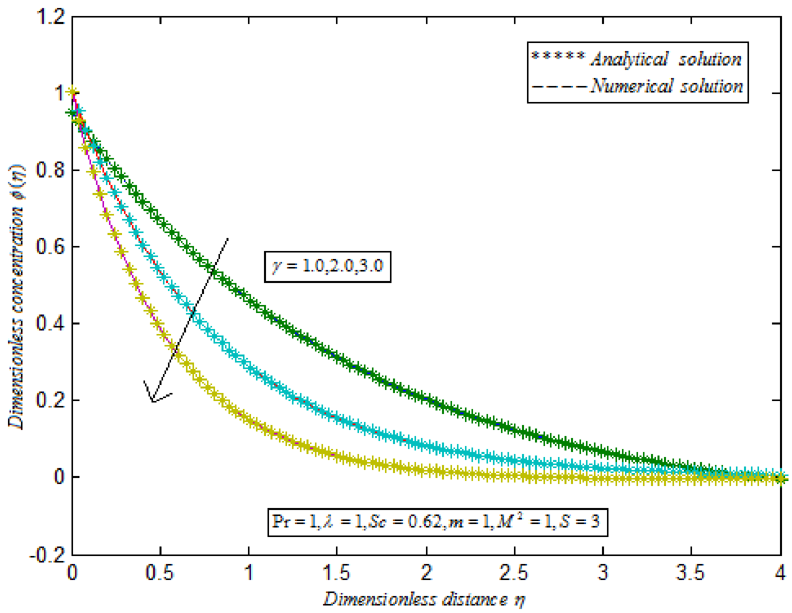

Fig.7: Dimensionless concentration $\phi(\eta)$ versus the dimensionless distance η . The curves are plotted using the eqn.(63) for various values of the Chemical reaction parameter γ, and in some fixed values of the other dimensionless parameters $\Pr, \lambda, Sc, M^2, m, S.$

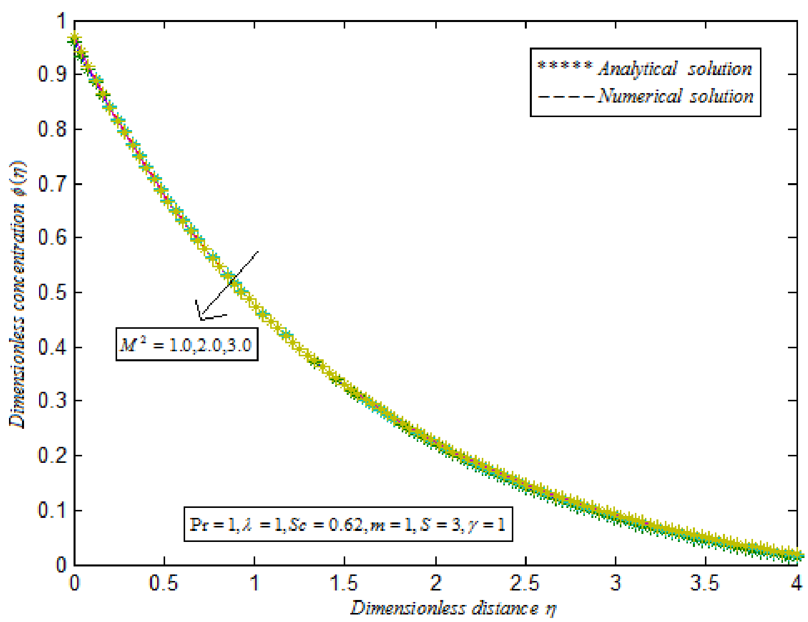

Fig.8: Dimensionless concentration $\phi(\eta)$ versus the dimensionless distance η . The curves are plotted using the eqn.(63) for various values of the Magnetic parameter M^2, and in some fixed values of the other dimensionless parameters $\Pr, \lambda, Sc, \gamma, m, S.$

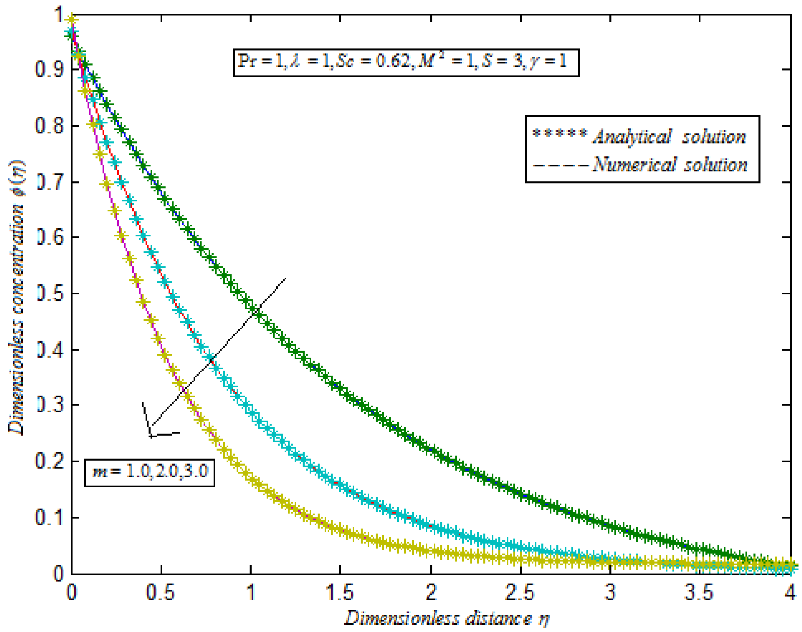

Fig.9: Dimensionless concentration $\phi(\eta)$ versus the dimensionless distance η . The curves are plotted using the eqn.(63) for various values of the Sheet shrinks parameter m, and in some fixed values of the other dimensionless parameters $\mathrm{Pr}, \lambda, Sc, \gamma, M^2, S$.

Table: 1 Comparison of our analytical results with the previous work

Previous work $f''(0)$	**Our work** $f''(0)$	**Error %**	**Dimensionless parameters**
3.302776	3.302751	0.00075	$\lambda = 1.0$
3.561553	3.561674	0.00339	$\lambda = 2.0$
4.000000	4.000874	0.02185	$\lambda = 4.0$
3.302776	3.302751	0.00075	$\gamma = 1.0$
3.302776	3.302751	0.00075	$\gamma = 2.0$
3.302776	3.302751	0.00075	$\gamma = 3.0$
3.302776	3.302890	0.00345	$M^2 = 1.0$
3.561533	3.561651	0.00331	$M^2 = 2.0$
3.791288	3.791335	0.00123	$M^2 = 3.0$
2.414214	2.413496	0.02974	$m = 1.0$
4.124816	4.124653	0.00395	$m = 2.0$
6.001955	6.000000	0.03257	$m = 3.0$

Table: 2 Comparison of our analytical results with the previous work

Previous work $\theta'(0)$	**Our work** $\theta'(0)$	**Error %**	**Dimensionless parameters**
-2.665537	-2.665600	0.00236	$\lambda = 1.0$
-2.680315	-2.680300	0.00055	$\lambda = 2.0$
-2.702455	-2.702500	0.00166	$\lambda = 4.0$
-2.665537	-2.665600	0.00236	$\gamma = 1.0$
-2.665537	-2.665600	0.00236	$\gamma = 2.0$
-2.665537	-2.665600	0.00236	$\gamma = 3.0$
-1.493292	-1.493200	0.00616	$m = 1.0$
-3.608226	-3.608377	0.00418	$m = 2.0$
-5.653797	-5.653839	0.00074	$m = 3.0$

Table: 3 Comparison of our analytical results with the previous work

Previous work $\phi'(0)$	**Our work** $\phi'(0)$	**Error %**	**Dimensionless parameters**
-2.410283	-2.410250	0.00136	$\lambda = 1.0$
-2.417000	-2.416998	0.00008	$\lambda = 2.0$
-2.427225	-2.427116	0.00449	$\lambda = 4.0$
-2.410283	-2.410250	0.00136	$\gamma = 1.0$
-2.929028	-2.929052	0.00081	$\gamma = 2.0$
-3.342858	-3.342900	0.00125	$\gamma = 3.0$
-1.917597	-1.917536	0.00318	$m = 1.0$
-2.865386	-2.865348	0.00132	$m = 2.0$
-3.944462	-3.944500	0.00096	$m = 3.0$

6. Conclusion

In this paper the Homotopy analysis method is employed to get the analytical expressions of the dimensionless velocity, dimensionless temperature, and dimensionless concentration for MHD fluid flow problem. In this work, also the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow past a porous shrinking sheet in the presence of suction is investigated. We conclude that the dimensionless concentration is always decreasing when magnetic parameter, chemical reaction parameter, and sheet shrinks parameter are increases. We also reported the error table for our analytical works and the numerical works (Previous works).

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